جديد

مجلة 5 min Maths

50 نهاية معقدة

مرفقة بالحل

المستوى

النهائي و الجامعي

جمع و كتابة : شعبان أسامة

أستاذ التعليم الثانوي

العدد الأول

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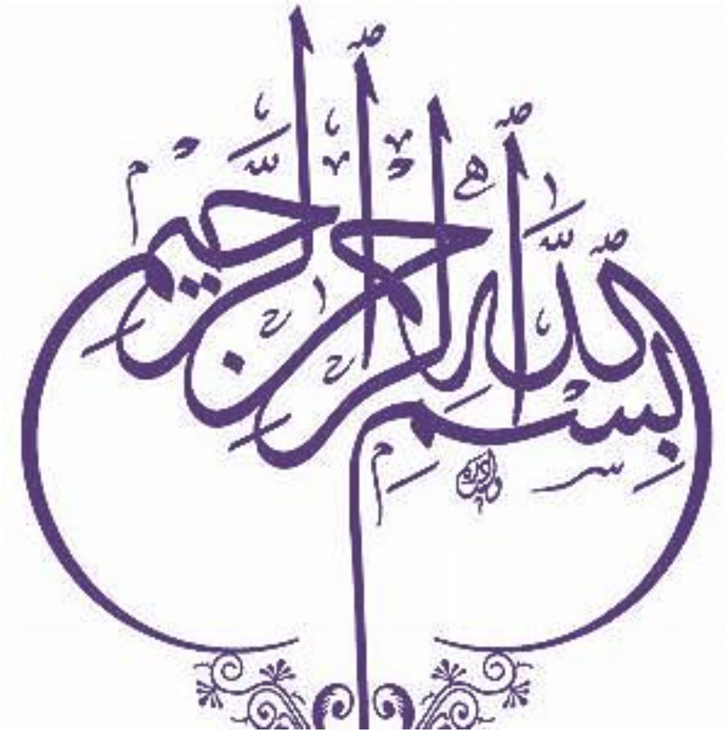

تعليق

بعد ملاحظتي لعدة منشورات في مواقع التواصل الاجتماعي تخص كيفية حل نهايات تبدو غير مالوفة و غير متداولة كثيرا. حاولت فبجمعها و ارفاقها بحلول مقترحة في هذا العمل المتواضع.

ففمنها كان عبارة عن اجتهاد شخصي و الشق الأخر هي محاولات الأساتذة (كرم مخلوف – عرفات الأسعدي) جزاهم الله كل خير

ف

فأهدي هذا العمل الى عائلتي و الى كل محب للرياضيات.

فاللهم ارحم من علمني

أ.شعبان

شملت هذه المجلة كل الحالات السبع المعرفة في الرياضيات

منهاج الثانوي:

$$\frac{0}{0}$$

$$\frac{\infty}{\infty}$$

$$0 \times \infty$$

$$\infty \pm \infty$$

$$1^{\infty}$$

$$0^{0}$$

$$\infty^{0}$$

نهاية 1: $\lim\limits_{x\to 0}\dfrac{e^x-x-1}{x^2}$.

نضع: $x^2=y$ تصبح:

$\lim\limits_{y\to 0}\dfrac{e^{\sqrt{y}}-\sqrt{y}-1}{y}$ بوضع :

$$g(y)=e^{\sqrt{y}}-\sqrt{y}$$

$$g'(y)=\frac{1}{2\sqrt{y}}e^{\sqrt{y}}-\frac{1}{2\sqrt{y}}=\frac{1}{2}\left(\frac{e^{\sqrt{y}}-1}{\sqrt{y}}\right)$$

أي:

$$\lim_{y\to 0}\frac{e^{\sqrt{y}}-\sqrt{y}-1}{y}=\lim_{y\to 0}\frac{g(y)-g(0)}{y-0}$$

$$=g'(0)=\frac{1}{2}$$

لأن: $\lim\limits_{y\to 0}\left(\dfrac{e^{\sqrt{y}}-1}{\sqrt{y}}\right)$.

اذن: $\lim\limits_{x\to 0}\dfrac{e^x-x-1}{x^2}=\dfrac{1}{2}$

نهاية 2: $\lim\limits_{x\to 1}\dfrac{x\sqrt{x+3}-2}{x^2+x-2}$

$$\lim_{x\to 1}\frac{x\sqrt{x+3}-2}{x^2+x-2}=\lim_{x\to 1}\frac{x\left(\sqrt{x+3}-\sqrt{4}\right)+2(x-1)}{x^2+x-2}$$

$$=\lim_{x\to 1}\frac{x\left(\sqrt{x+3}-\sqrt{4}\right)}{(x-2)(x-1)}+\frac{2(x-1)}{(x-2)(x-1)}$$

$$=\frac{3}{4}$$

أي: $\lim\limits_{x\to 1}\dfrac{x\sqrt{x+3}-2}{x^2+x-2}=\dfrac{3}{4}$.

نهاية 3: $\lim\limits_{x\to +\infty}\left(\dfrac{4^{\frac{1}{x}}+5^{\frac{1}{x}}+6^{\frac{1}{x}}}{3}\right)^{3x}$

نعلم أن:

$$\lim_{x\to 0}\frac{a^x-1}{x}=\ln a$$

$$\lim_{x\to 1}\frac{\ln x}{x-1}=1$$

بوضع: $y=\dfrac{1}{x}\Leftrightarrow y\to 0$

$$\lim_{y\to 0}\left(\frac{4^y+5^y+6^y}{3}\right)^{\frac{3}{y}}=e^{\lim\limits_{y\to 0}\frac{3}{y}\ln\left(\frac{4^y+5^y+6^y}{3}\right)}$$

$$=\lim_{y\to 0}\frac{3}{y}\frac{\ln\left(\frac{4^y+5^y+6^y}{3}\right)}{\left(\frac{4^y+5^y+6^y}{3}\right)-1}.\left(\left(\frac{4^y+5^y+6^y}{3}\right)-1\right)$$

$$=\lim_{y\to 0}1.\frac{3}{y}\left(\frac{4^y+5^y+6^y-3}{3}\right)$$

$$=\lim_{y\to 0}\left(\frac{4^y-1}{y}+\frac{5^y-1}{y}+\frac{6^y-1}{y}\right)$$

$$=\ln 4+\ln 5+\ln 6$$

$$=\ln 120$$

و بالتالي: $\lim\limits_{x\to +\infty}\left(\dfrac{4^{\frac{1}{x}}+5^{\frac{1}{x}}+6^{\frac{1}{x}}}{3}\right)^{3x}=\ln(120)$.

نهاية 4: $\lim\limits_{n\to +\infty}\left(\dfrac{n+9}{n+10}\right)^n$

$$\lim_{n\to +\infty}\left(\frac{n+9}{n+10}\right)^n=e^{\lim\limits_{n\to +\infty} n\left(\frac{n+9}{n+10}\right)}$$

$$e^{\lim\limits_{n\to +\infty} n\frac{\left(\frac{n+9}{n+10}\right)}{\left(\frac{n+9}{n+10}\right)-1}.\left(\left(\frac{n+9}{n+10}\right)-1\right)}$$

$$e^{\lim\limits_{n\to +\infty} n.1.\left(\left(\frac{n+9}{n+10}\right)-1\right)}$$

$$e^{\lim\limits_{n\to +\infty} n.\left(\frac{n+9-n-10}{n+10}\right)}$$

$$e^{-1}=\frac{1}{e}$$

و بالتالي: $\lim\limits_{n\to +\infty}\left(\dfrac{n+9}{n+10}\right)^n=\dfrac{1}{e}$

نهاية 5: $\lim\limits_{x\to 1}\dfrac{(2x+1)^5-243}{(x+1)^6-32}$.

نلاحظ أن : $x-1=(x+1)=\left((2x+1)-3\right).\dfrac{1}{2}$

و منه:

$$\lim_{x\mapsto 1}\frac{(2x+1)^5-243}{(x+1)^6-32}=\lim_{x\mapsto 1}\frac{2.\dfrac{(2x+1)^5-3^5}{(2x+1)^1-3^1}}{\dfrac{(x+1)^6-2^5}{(x+1)^1-2^1}}$$

$$=2.\frac{\frac{5}{1}(3)^4}{\frac{6}{1}(2)^5}=\frac{135}{32}$$

اذن: $\lim_{x\mapsto 1}\frac{(2x+1)^5-243}{(x+1)^6-32}=\frac{135}{32}$.

نهاية 6: $\lim_{n\mapsto+\infty}\sqrt[n]{4^n+5^n}$.

$$\lim_{n\mapsto+\infty}\sqrt[n]{4^n+5^n}=\lim_{n\mapsto+\infty}\left(4^n+5^n\right)^{\frac{1}{n}}$$

$$=\lim_{n\mapsto+\infty}\left(5^n\right)^{\frac{1}{n}}.\left(\left(\frac{4}{5}\right)^n+1\right)^{\frac{1}{n}}$$

$$=5.\lim_{n\mapsto+\infty}\left(1+\left(\frac{4}{5}\right)^n\right)^{\frac{1}{n}}$$

$$=5.e^{\lim_{n\mapsto+\infty}\left(\frac{4}{5}\right)^n.\frac{1}{n}}=5.e^{0.0}=5$$

اذن: $\lim_{n\mapsto+\infty}\sqrt[n]{4^n+5^n}=5$.

نهاية 7: $\lim_{x\mapsto 1}\frac{\sin(\pi x)+\cos\left(\frac{\pi x}{2}\right)}{(1+3x).\cos(\frac{3\pi x}{2})}$.

نعلم أن: $\lim_{x\mapsto a}\frac{\sin(bx)}{\sin(dx)}=\frac{b}{d}$.

$$\lim_{x\mapsto 1}\frac{\sin(\pi x)+\cos\left(\frac{\pi x}{2}\right)}{(1+3x).\cos(\frac{3\pi x}{2})}=\lim_{x\mapsto 1}\frac{\sin(\pi x)+\sin\left(\frac{\pi}{2}-\frac{\pi x}{2}\right)}{(1+3x).\sin\left(\frac{\pi}{2}-\frac{\pi x}{2}\right)}$$

$$=\lim_{x\mapsto 1}\frac{\sin(\pi x)}{(1+3x).\sin\frac{\pi}{2}(1-3x)}+\lim_{x\mapsto 1}\frac{\sin\frac{\pi}{2}(1-x)}{(1+3x).\sin\frac{\pi}{2}(1-3x)}$$

$$=\lim_{x\mapsto 1}\frac{\pi x}{(1+3x).\sin\frac{\pi}{2}(1-3x)}+\lim_{x\mapsto 1}\frac{\frac{\pi}{2}(1-x)}{(1+3x).\sin\frac{\pi}{2}(1-3x)}$$

$$=\frac{-2}{2(4)}-\frac{0}{2(4)}=\frac{-1}{4}$$

اذن: $\lim_{x\mapsto 1}\frac{\sin(\pi x)+\cos\left(\frac{\pi x}{2}\right)}{(1+3x).\cos(\frac{3\pi x}{2})}=\frac{-1}{4}$.

نهاية 8: $\lim_{x\mapsto\frac{\pi}{2}}\frac{\cos}{\cos(3x)}$.

نعلم أن: $\cos(3x)=4\cos^3(x)-3\cos(x)$.

$$\lim_{x\mapsto\frac{\pi}{2}}\frac{\cos(x)}{\cos(3x)}=\lim_{x\mapsto\frac{\pi}{2}}\frac{\cos(x)}{4\cos^3(x)-3\cos(x)}$$

$$=\lim_{x\mapsto\frac{\pi}{2}}\frac{\cos(x)}{\cos(x)\left(4\cos^2(x)-3\right)}$$

$$=\lim_{x\mapsto\frac{\pi}{2}}\frac{1}{\left(4\cos^2(x)-3\right)}$$

$$=\frac{-1}{3}$$

نهاية 9: $\lim_{x\mapsto+\infty}\sqrt[x]{x}$

نعلم أن: $\lim_{x\mapsto 0}x\ln(x)=0$.

نضع: $y=\frac{1}{x}\Leftrightarrow y\to 0$.

$$\lim_{x\mapsto+\infty}\sqrt[x]{x}=\lim_{x\mapsto 0^+}\left(\frac{1}{y}\right)^y=\frac{1}{\lim_{x\mapsto 0^+}(y)^y}$$

حيث:

$$\lim_{x\mapsto 0^+}(y)^y=e^{\lim_{x\mapsto 0^+}y\ln(y)}$$

$$=e^0=1$$

و بالتالي: $\lim_{x\mapsto+\infty}\sqrt[x]{x}=1$.

نهاية 10: $\lim\limits_{x\to0}\frac{\left(2-x^2\right)\sin\left(x\right)-\sin\left(2x\right)}{x^5}$.

نعلم أن: $\lim\limits_{x\to0}\frac{\sin\left(x\right)-x}{x^3}=\frac{-1}{6}$.

$$\lim_{x\to0}\frac{\left(2-x^2\right)\sin\left(x\right)-\sin\left(2x\right)}{x^5}$$
$$=\lim_{x\to0}\frac{2\sin\left(x\right)-x^2\sin(x)-2\sin\left(x\right).\cos(x)}{x^5}$$
$$=\lim_{x\to0}\frac{2\sin\left(x\right)(1-\cos(x)-x^2\sin(x)}{x^5}$$
$$=\lim_{x\to0}\frac{\sin x}{x}.\frac{2(2)\sin^2\left(\frac{x}{2}\right)-x^2}{x^4}$$
$$=1.\lim_{x\to0}\frac{4\sin^2\left(\frac{x}{2}\right)-x^2}{x^4}$$
$$=\lim_{x\to0}\frac{\left(2\sin^2\left(\frac{x}{2}\right)+x\right)}{x}.\frac{\left(2\sin^2\left(\frac{x}{2}\right)-x\right)}{x^3}$$
$$=\lim_{x\to0}\left(\frac{2\sin^2\left(\frac{x}{2}\right)}{x}+\frac{x}{x}\right).\frac{\left(2\sin^2\left(\frac{x}{2}\right)-\frac{2x}{2}\right)}{x^3}$$
$$=\left(2.\left(\frac{1}{2}\right)+1\right).2\lim_{x\to0}\frac{\sin\left(\frac{x}{2}\right)-\left(\frac{x}{2}\right)}{\left(\frac{x}{2}\right)^3}.\frac{\left(\frac{x}{2}\right)^3}{x^3}$$
$$=2.2.\frac{-1}{6}.\frac{1}{8}=\frac{-1}{12}$$

و بالتالي: $\lim\limits_{x\to0}\frac{\left(2-x^2\right)\sin\left(x\right)-\sin\left(2x\right)}{x^5}=-\frac{1}{12}$.

نهاية 11: $\lim\limits_{x\to0}\tan\left(\frac{\pi}{4}+x\right)^{\frac{1}{x}}$

$$\lim_{x\to0}\tan\left(\frac{\pi}{4}+x\right)^{\frac{1}{x}}=\lim_{x\to0}\left(1+\tan\left(\frac{\pi}{4}+x\right)-1\right)^{\frac{1}{x}}$$
$$=e^{\lim\limits_{x\to0}\frac{\tan\left(\frac{\pi}{4}+x\right)-1}{x}}=e^{\lim\limits_{x\to0}\frac{\tan\left(\frac{\pi}{4}+x\right)-\tan\left(\frac{\pi}{4}\right)}{x}}$$
$$=e^{\lim\limits_{x\to0}\frac{\tan\left(\frac{\pi}{4}+x-\frac{\pi}{4}\right)\left(1+\tan\left(\frac{\pi}{4}+x\right)\right).\tan\left(\frac{\pi}{4}\right)}{x}}$$
$$e^{\lim\limits_{x\to0}\frac{\tan x}{x}.\left(1+\tan\left(\frac{\pi}{4}+x\right)\right)}$$
$$=e^2$$

و بالتالي: $\lim\limits_{x\to0}\tan\left(\frac{\pi}{4}+x\right)^{\frac{1}{x}}=e^2$.

نهاية 12 : $\lim\limits_{x\to+\infty}\left(\cos\left(\frac{1}{x}\right)+\sin\left(\frac{1}{x^2}\right)\right)^{x^2}$

نعلم أن: $\lim\limits_{x\to0}\frac{\ln(1+x)}{x}=1$.

نضع: $x=\frac{1}{y}, x\to+\infty, y\to0$

$$\lim_{x\to+\infty}\left(\cos\left(\frac{1}{x}\right)+\sin\left(\frac{1}{x^2}\right)\right)^{x^2}$$
$$=\lim_{y\to0}\left(\cos\left(y\right)+\sin\left(y^2\right)\right)^{\frac{1}{y^2}}$$
$$=e^{\lim\limits_{y\to0}\frac{1}{y^2}\ln\left(\cos(y)+\sin\left(y^2\right)\right)}$$
$$=e^{\lim\limits_{y\to0}\frac{\ln\left(\cos(y)+\sin\left(y^2\right)\right)}{y^2}}$$
$$=e^{\lim\limits_{y\to0}\frac{\ln\left(\cos(y)+\sin\left(y^2\right)\right)}{\left(\cos(y)+\sin\left(y^2\right)-1\right)}.\frac{\left(\cos(y)+\sin\left(y^2\right)-1\right)}{y^2}}$$
$$=e^{\lim\limits_{y\to0}.\frac{\left(\cos(y)+\sin\left(y^2\right)-1\right)}{y^2}}$$
$$=e^{\lim\limits_{y\to0}.\frac{-2\sin\left(\frac{y^2}{2}\right)+\sin\left(y^2\right)}{y^2}}$$
$$=e^{-2.\frac{1}{4}+1}=e^{\frac{1}{2}}=\sqrt{e}$$

و بالتالي: $\lim\limits_{x\to+\infty}\left(\cos\left(\frac{1}{x}\right)+\sin\left(\frac{1}{x^2}\right)\right)^{x^2}=\sqrt{e}$.

نهاية 13 : $\lim\limits_{x\mapsto 2^-}\frac{x-2\cos(\sqrt{2-x})}{(2-x)^2}$

نعلم أن: $\lim\limits_{x\mapsto 0}\frac{\sin(x)-x}{x^3}=\frac{-1}{6}$

نضع: $y=2-x, x\to 2^-, y\to 0$

$$\lim_{x\mapsto 2^-}\frac{x-2\cos(\sqrt{2-x})}{(2-x)^2}$$

$$=\lim_{y\mapsto 0}\frac{2-y-2\cos(\sqrt{y})}{y^2}$$

$$=\lim_{y\mapsto 0}\frac{2\left(1-\cos(\sqrt{y})\right)-y}{y^2}$$

$$=\lim_{y\mapsto 0}\frac{2\left(2\sin^2(\frac{\sqrt{y}}{2})\right)-y}{y^2}$$

$$=\lim_{y\mapsto 0}\frac{1}{16}\frac{4\sin^2(\frac{\sqrt{y}}{2})-(\frac{\sqrt{y}}{2})^2}{y^4}$$

$$=\frac{4}{16}\lim_{y\mapsto 0}\frac{\sin(\frac{\sqrt{y}}{2})+(\frac{\sqrt{y}}{2})}{y}\times\frac{\sin(\frac{\sqrt{y}}{2})-(\frac{\sqrt{y}}{2})}{y^3}$$

$$=\frac{1}{4}.2.\frac{-1}{6}=\frac{-1}{12}$$

و منه: $\lim\limits_{x\mapsto 2^-}\frac{x-2\cos(\sqrt{2-x})}{(2-x)^2}=\frac{-1}{12}$.

نهاية 14 : $\lim\limits_{n\mapsto+\infty}\frac{1}{n^2}+\frac{2}{n^2}+...+\frac{n-1}{n^2}$.

نعلم أن : $1+2+3+...+(n-1)=\frac{n(n-1)}{2}$.

$$\lim_{n\mapsto+\infty}\frac{1}{n^2}+\frac{2}{n^2}+...+\frac{n-1}{n^2}$$

$$=\lim_{n\mapsto+\infty}\frac{1}{n^2}\left(1+2+3+...+(n-1)\right)$$

و بالتالي:

$$=\lim_{n\mapsto+\infty}\frac{1}{n^2}\left(\frac{n(n-1)}{2}\right)$$

$$=\lim_{n\mapsto+\infty}\frac{n^2-n}{2n^2}=\frac{1}{2}$$

نهاية 15 : $\lim\limits_{x\to\frac{\pi}{2}}(\sin x)^{\frac{1}{\cos x}}$

نعلم أن: $\lim\limits_{x\mapsto 1}\frac{\ln x}{x-1}=1$.

$$\lim_{x\mapsto\frac{\pi}{2}}(\sin x)^{\frac{1}{\cos x}}=e^{\lim\limits_{x\mapsto\frac{\pi}{2}}\frac{1}{\cos x}\ln(\sin x)}$$

$$=e^{\lim\limits_{x\mapsto\frac{\pi}{2}}\frac{\ln(\sin x)}{\cos x}}=e^{\lim\limits_{x\mapsto\frac{\pi}{2}}\frac{\ln(\sin x)}{\sin x-1}\times\frac{\sin x-1}{\cos x}}$$

$$=e^{1\times\lim\limits_{x\mapsto\frac{\pi}{2}}\frac{(\sin x-1)}{(\sin x+1)}\times\frac{(\sin x+1)}{\cos x}}$$

$$=e^{\lim\limits_{x\mapsto\frac{\pi}{2}}\frac{\cos^2 x}{\cos x(\sin x+1)}}$$

$$=e^{\frac{0}{2}}=1$$

اذن: $\lim\limits_{x\to\frac{\pi}{2}}(\sin x)^{\frac{1}{\cos x}}=1$.

نهاية 16 : $\lim\limits_{x\mapsto 0}\frac{1-\cos x\sqrt{\cos 2x}}{x^2}$.

$$\lim_{x\mapsto 0}\frac{1-\cos x\sqrt{\cos 2x}}{x^2}=\lim_{x\mapsto 0}\frac{1-\sqrt{\cos^2 x.\cos 2x}}{x^2}$$

$$\lim_{x\mapsto 0}\frac{1-\sqrt{\cos^2 x.\cos 2x}}{1-\left(\cos^2 x.\cos x\right)}\times\frac{1-\left(\cos^2 x.\cos x\right)}{x^2}$$

$$=\frac{1}{2}(1)^{\frac{1}{2}-1}\times\lim_{x\mapsto 0}\frac{1-\left(1-\sin^2 x\right).\cos 2x}{x^2}$$

$$= \frac{1}{2}\lim_{x\mapsto0}\frac{1-.\cos 2x+.\cos 2x\left(\sin^2 x\right)}{x^2}$$

$$= \frac{1}{2}\lim_{x\mapsto0}\left(\frac{2\left(\sin^2 x\right)}{x^2}+\frac{\left(\sin^2 x\right)}{x^2}\cos 2x\right)$$

$$= \frac{1}{2}(2+1\times1)=\frac{3}{2}$$

و بالتالي: $\lim_{x\mapsto0}\frac{1-\cos x\sqrt{\cos 2x}}{x^2}=\frac{3}{2}$.

نهاية 17: $\lim_{x\to+\infty}\frac{e^x}{\ln x}$.

$$\lim_{x\to+\infty}\frac{e^x}{\ln x}=\lim_{x\to+\infty}\left(\frac{e^x}{x}\right)\times\left(\frac{x}{\ln x}\right)=+\infty .$$

نهاية 18 : $\lim_{x\to+\infty}\left(\frac{x+2}{x}\right)^x$.

$$\lim_{x\to+\infty}\left(\frac{x+2}{x}\right)^x=e^{\lim_{x\to+\infty} x\ln\left(\frac{x+2}{x}\right)}$$

$$=e^{\lim_{x\to+\infty} x\ln\left(1+\frac{2}{x}\right)}=e^{\lim_{x\to+\infty} 2\frac{\ln\left(1+\frac{2}{x}\right)}{\frac{x}{2}}}$$

نضع: $t=\frac{2}{x}, x\to+\infty, t\to0$.

$$e^{\lim_{x\to+\infty} 2\frac{\ln\left(1+\frac{2}{x}\right)}{\frac{x}{2}}}=e^{\lim_{x\to+\infty} 2\frac{\ln(1+t)}{t}}=e^2$$

و بالتالي: $\lim_{x\to+\infty}\left(\frac{x+2}{x}\right)^x=e^2$.

نهاية 19: $\lim_{x\mapsto0} x^{\ln\left(1+\frac{x}{\ln x}\right)}$

$$\lim_{x\mapsto0} x^{\ln\left(1+\frac{x}{\ln x}\right)}=\lim_{x\mapsto0} e^{\ln\left(x^{\ln\left(1+\frac{x}{\ln x}\right)}\right)}$$

$$=\lim_{x\mapsto0} e^{\ln\left(x^1\right)\ln\left(1+\frac{x}{\ln x}\right)}$$

نعلم أن: $\lim_{x\mapsto0}\frac{\ln(1+x)}{x}=1$

$$=\lim_{x\mapsto0}\ln(x)\ln\left(1+\frac{x}{\ln x}\right)$$

و بالتالي:

$$=\lim_{x\mapsto0}\ln(x)\frac{\ln\left(1+\frac{x}{\ln x}\right)}{\frac{x}{\ln x}}.\frac{x}{\ln x}$$

$$=\lim_{x\mapsto0}\ln(x).(1).\frac{x}{\ln x}$$

$$=0$$

و منه: $\lim_{x\mapsto0} x^{\ln\left(1+\frac{x}{\ln x}\right)}=e^0=1$

اذن: $\lim_{x\mapsto0} x^{\ln\left(1+\frac{x}{\ln x}\right)}=1$

نهاية 20: $\lim_{n\mapsto+\infty} e^{-x}\sin(e^x)$.

$$\lim_{n\mapsto+\infty} e^{-x}\sin(e^x)=\lim_{n\mapsto+\infty}\frac{\sin(e^x)}{e^x}$$

$$\lim_{n\mapsto+\infty}\frac{1}{e^x}\to0$$

و لدينا: $\left|\sin(e^x)\right|\le1$

و نعلم أن نهاية دالة صفرية في دالة محدودة تساوي 0

اذن: $\lim_{n\mapsto+\infty} e^{-x}\sin(e^x)=0$

نهاية 21: $\lim_{x\mapsto0}\frac{1-\cos(\sin x)}{x^2}$

$$\lim_{x\mapsto0}\frac{1-\cos(\sin x)}{x^2}$$

$$=\lim_{x\mapsto0}\frac{1-\cos(\sin x)}{x^2}.\frac{1+\cos(\sin x)}{1+\cos(\sin x)}$$

$$=\lim_{x\mapsto0}\frac{1-\cos^2(\sin x)}{x^2}.\frac{1}{1+\cos(\sin x)}$$

$$= \lim_{x\to 0} \frac{\sin^2(\sin x)}{x^2}\cdot\frac{\sin^2(x)}{\sin^2(x)}\cdot\frac{1}{2}$$

$$= \lim_{x\to 0} \frac{\sin^2(\sin x)}{\sin^2(x)}\cdot\frac{\sin^2(x)}{x^2}\cdot\frac{1}{2}$$

$$= \frac{1}{2}$$

إذن: $\lim_{x\to 0} \frac{1-\cos(\sin x)}{x^2} = \frac{1}{2}$

نهاية 22: $\lim_{x\mapsto 1} \frac{1}{\ln x} - \frac{1}{1-x}$.

نعلم أن :

$$\lim_{x\mapsto 1} \frac{e^x - x - 1}{x^2} = \frac{1}{2}$$

$$\lim_{x\mapsto 1} \frac{e^x - 1}{x} = 1$$

نضع: $x = e^y$.

$$\lim_{x\mapsto 1} \frac{1}{\ln x} - \frac{1}{1-x} = \lim_{x\mapsto 1} \frac{1-x-\ln x}{(1-x)\ln x}$$

$$= \lim_{y\mapsto 0} \left(\frac{-(e^y - y - 1)}{\left(1-e^y\right).y} \right)$$

$$= \lim_{y\mapsto 0} \frac{-(e^y - y - 1)}{y^2}\cdot\frac{y^2}{\left(1-e^y\right).y}$$

$$= \frac{-1}{2}\times -1$$

$$= \frac{1}{2}$$

و بالتالي: $\lim_{x\mapsto 1} \frac{1}{\ln x} - \frac{1}{1-x} = \frac{1}{2}$

نهاية 23: $\lim_{n\mapsto\infty} \frac{\tan\left(\frac{1}{x}+\frac{\pi}{2}\right)}{x}$

$$\lim_{x\mapsto\infty} \frac{\tan\left(\frac{1}{x}+\frac{\pi}{2}\right)}{x} = \lim_{x\mapsto\infty} \frac{\cot\left(-\frac{1}{x}\right)}{x}$$

$$= -1\times\lim_{x\mapsto\infty} \frac{\cot\left(\frac{1}{x}\right)}{x}$$

$$= -1\times\lim_{x\mapsto\infty} \frac{1}{x\tan\left(\frac{1}{x}\right)}$$

$$= -1\times\lim_{x\mapsto\infty} \frac{\frac{1}{x}}{\tan\left(\frac{1}{x}\right)} = -1$$

اذن : $\lim_{x\mapsto\infty} \frac{\tan\left(\frac{1}{x}+\frac{\pi}{2}\right)}{x} = -1$.

نهاية 24 : $\lim_{x\mapsto\frac{\pi}{2}} \frac{1-\sin x}{\frac{\pi}{2}-x}$

نعلم أن:

$$1-\sin x = sin\frac{\pi}{2} - sinx = 2\cos\left(\frac{\pi}{4}+\frac{x}{2}\right).\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)$$

$$\lim_{x\mapsto\frac{\pi}{2}} \frac{1-\sin x}{\frac{\pi}{2}-x} = \lim_{x\mapsto\frac{\pi}{2}} \frac{2\cos\left(\frac{\pi}{4}+\frac{x}{2}\right).\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)}{\frac{\pi}{2}-x}$$

$$= 4\cos\left(\frac{\pi}{2}\right).\lim_{x\mapsto\frac{\pi}{2}} \frac{\sin\frac{1}{4}(\pi-2x)}{\frac{1}{4}(\pi-2x)}\cdot\frac{1}{4}$$

$$= 0$$

اذن: $\lim_{x\mapsto\frac{\pi}{2}} \frac{1-\sin x}{\frac{\pi}{2}-x} = 0$.

نهاية 25: $\lim_{x\mapsto 0} \frac{x-\sin^{-1} x}{x-\sin x}$

نضع : $x = \sin y$

$y = \sin^{-1} x$

$$\lim_{x\to 0}\frac{x-\sin^{-1}x}{x-\sin x}=\lim_{y\to 0}\frac{\sin y-y}{\sin y-\sin(\sin y)}$$

نعلم أن:

$$\sin y-\sin(\sin y)=2\cos\left(\frac{y+\sin y}{2}\right)\sin\left(\frac{y-\sin y}{2}\right)$$

$$\lim_{y\to 0}\frac{\sin y-y}{\sin y-\sin(\sin y)}$$

$$=\lim_{y\to 0}\frac{\sin y-y}{2\cos\left(\frac{y+\sin y}{2}\right)\sin\left(\frac{y-\sin y}{2}\right)}$$

$$=\lim_{y\to 0}\frac{\frac{\sin y-y}{2}}{\cos\left(\frac{y+\sin y}{2}\right)\sin\left(\frac{y-\sin y}{2}\right)}$$

$$=\lim_{y\to 0}\frac{\frac{\sin y-y}{2}}{\sin\left(\frac{y-\sin y}{2}\right)}\times\lim_{y\to 0}\frac{1}{\cos\left(\frac{y+\sin y}{2}\right)}$$

$$=-1\lim_{y\to 0}\frac{\frac{y-\sin y}{2}}{\sin\left(\frac{y-\sin y}{2}\right)}\times\lim_{y\to 0}\frac{1}{\cos\left(\frac{y+\sin y}{2}\right)}$$

$$=-1.1.1=-1$$

اذن: $\lim_{x\to 0}\frac{x-\sin^{-1}x}{x-\sin x}=-1$.

نهاية 26: $\lim_{x\to 0^+}\frac{\sin\left(\sqrt{\sin\pi x}\right)}{x-\sqrt{x}}$.

$$\lim_{x\to 0^+}\frac{\sin\left(\sqrt{\sin\pi x}\right)}{x-\sqrt{x}}=\lim_{x\to 0^+}\frac{\sin\left(\sqrt{\sin\pi x}\right)}{x-\sqrt{x}}.\frac{\sqrt{\sin\pi x}}{\sqrt{\sin\pi x}}$$

$$=\lim_{x\to 0^+}\frac{\sin\left(\sqrt{\sin\pi x}\right)}{\sqrt{\sin\pi x}}\times\lim_{x\to 0^+}\frac{\sqrt{\sin\pi x}}{x-\sqrt{x}}\times\frac{\sqrt{\pi x}}{\sqrt{\pi x}}$$

$$=1\times\lim_{x\to 0^+}\frac{\sin\left(\sqrt{\sin\pi x}\right)}{\sqrt{\pi x}}\times\lim_{x\to 0^+}\frac{\sqrt{\pi x}}{x-\sqrt{x}}$$

$$=\lim_{x\to 0^+}\frac{\sin\left(\sqrt{\sin\pi x}\right)}{\sqrt{\pi x}}\times\lim_{x\to 0^+}\frac{\sqrt{\pi x}}{\sqrt{x}\left(\sqrt{x}-1\right)}$$

$$=1.\lim_{x\to 0^+}\frac{\sqrt{\pi}.\sqrt{x}}{\sqrt{x}\left(\sqrt{x}-1\right)}$$

$$=\lim_{x\to 0^+}\frac{\sqrt{\pi}.}{\left(\sqrt{x}-1\right)}$$

$$=-\sqrt{\pi}$$

اذن: $\lim_{x\to 0^+}\frac{\sin\left(\sqrt{\sin\pi x}\right)}{x-\sqrt{x}}=-\sqrt{\pi}$.

نهاية 27: $a>0,\lim_{x\to a}\frac{\sqrt{x}-\sqrt{a}+\sqrt{x-a}}{\sqrt{x^2-a^2}}$

$$\lim_{x\to a}\frac{\sqrt{x}-\sqrt{a}+\sqrt{x-a}}{\sqrt{x^2-a^2}}=\lim_{x\to a}\frac{\sqrt{x}-\sqrt{a}+\sqrt{x-a}}{\sqrt{(x+a)(x-a)}}$$

$$=\lim_{x\to a}\frac{\sqrt{x-a}\left(\frac{\sqrt{x}-\sqrt{a}}{\sqrt{x-a}}+1\right)}{\left(\sqrt{x+a}\right)\left(\sqrt{x-a}\right)}$$

$$=\lim_{x\to a}\frac{\left(\frac{\sqrt{x}-\sqrt{a}}{\sqrt{x-a}}+1\right)}{\left(\sqrt{x+a}\right)}$$

$$=\lim_{x\to a}\frac{1}{\left(\sqrt{x+a}\right)}\times\lim_{x\to a}\left(\frac{\sqrt{x}-\sqrt{a}}{\sqrt{x-a}}+1\right)$$

$$=\frac{1}{\sqrt{2a}}\times\lim_{x\to a}\left(\frac{\sqrt{x}-\sqrt{a}}{\sqrt{x-a}}\times\frac{\sqrt{x-a}}{\sqrt{x-a}}+1\right)$$

$$=\frac{1}{\sqrt{2a}}\times\lim_{x\to a}\left(\frac{\left(\sqrt{x}-\sqrt{a}\right)\sqrt{x-a}}{x-a}+1\right)$$

$$=\frac{1}{\sqrt{2a}}\times\lim_{x\to a}\left(\frac{\left(\sqrt{x}-\sqrt{a}\right)\sqrt{x-a}}{\left(\sqrt{x}-\sqrt{a}\right)\left(\sqrt{x}+\sqrt{a}\right)}+1\right)$$

$$=\frac{1}{\sqrt{2a}}\times\lim_{x\to a}\left(\frac{\sqrt{x-a}}{\left(\sqrt{x}+\sqrt{a}\right)}+1\right)$$

$$= \frac{1}{\sqrt{2a}} \times (0+1)$$

$$= \frac{1}{\sqrt{2a}}$$

اذن: $a > 0, \lim_{x \mapsto a} \frac{\sqrt{x}-\sqrt{a}+\sqrt{x-a}}{\sqrt{x^2-a^2}} = \frac{1}{\sqrt{2a}}$.

نهاية 28: $\lim_{x \mapsto \pi} \frac{1-\sin\frac{x}{2}}{x-\pi}$.

نعلم أن:

$$1+\sin x = 2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)$$

$$1-\sin x = 2\sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)$$

$$\lim_{x \mapsto \pi} \frac{1-\sin\frac{x}{2}}{x-\pi} = \lim_{x \mapsto \pi} \frac{2\sin^2\left(\frac{\pi}{4}-\frac{x}{4}\right)}{4\left(\frac{x}{4}-\frac{\pi}{4}\right)}$$

و بالتالي:

$$= \lim_{x \mapsto \pi} \frac{2\sin\left(\frac{\pi}{4}-\frac{x}{4}\right)}{-4\left(\frac{\pi}{4}-\frac{x}{4}\right)} \times \sin\left(\frac{\pi}{4}-\frac{x}{4}\right)$$

$$= \frac{-1}{2}.0 = 0$$

اذن: $\lim_{x \mapsto \pi} \frac{1-\sin\frac{x}{2}}{x-\pi} = 0$.

نهاية 29: $\lim_{x \mapsto +\infty} x^2 \ln\left(\frac{x+1}{x}\right) - x$

$$\lim_{x \mapsto +\infty} x^2 \ln\left(\frac{x+1}{x}\right) - x = \lim_{x \mapsto +\infty} \frac{\ln\left(\frac{x+1}{x}\right)}{\frac{1}{x^2}} - x$$

نضع: $t = \frac{1}{x^2}$ اذن: $t \to 0$.

$$\lim_{t \mapsto 0} \frac{\ln\left(1+\sqrt{t}\right)}{t} - \frac{1}{\sqrt{t}}$$

$$= \lim_{t \mapsto 0} \frac{\ln\left(1+\sqrt{t}\right)-\sqrt{t}-0}{t-0}$$

نهاية العدد المشتق حيث: $f(t) = \ln\left(1+\sqrt{t}\right) - \sqrt{t}$.

$$\lim_{t \mapsto 0} \frac{\ln\left(1+\sqrt{t}\right)-\sqrt{t}-0}{t-0} = f'(0) = \frac{-1}{2}.$$

اذن: $\lim_{x \mapsto +\infty} x^2 \ln\left(\frac{x+1}{x}\right) - x = \frac{-1}{2}$.

نهاية 30: $\lim_{x \mapsto \frac{\pi}{4}} \frac{4\sqrt{2}-(\cos x+\sin x)^5}{1-2\sin x}$

$$\lim_{x \mapsto \frac{\pi}{4}} \frac{4\sqrt{2}-(\cos x+\sin x)^5}{1-2\sin x}$$

$$= \lim_{x \mapsto \frac{\pi}{4}} \frac{4\sqrt{2}-4\sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x+\frac{1}{\sqrt{2}}\sin x\right)^5}{1-\cos\left(\frac{\pi}{2}-2x\right)}$$

$$= \lim_{x \mapsto \frac{\pi}{4}} \frac{4\sqrt{2}\left(1-\cos^5\left(\frac{\pi}{4}-x\right)\right)}{1-2\cos^2\left(\frac{\frac{\pi}{2}-2x}{2}\right)-1}$$

$$= \lim_{x \mapsto \frac{\pi}{4}} \frac{4\sqrt{2}\left(1-\cos^5\left(\frac{\pi}{4}-x\right)\right)}{2\cos^2\left(\frac{\frac{\pi}{2}-2x}{2}\right)}$$

$$= \frac{4\sqrt{2}}{2}.\frac{5}{2}.(1)^{5-2} = 5\sqrt{2}.$$

إذن: $\lim_{x\to\frac{\pi}{4}} \frac{4\sqrt{2}-(\cos x+\sin x)^5}{1-2\sin x} = 5\sqrt{2}$.

نهاية 31: $\lim_{x\to\frac{\pi}{2}} (\sin x)^{\tan x}$.

نضع: $y=\frac{\pi}{2}-x$.

$$\lim_{y\to 0}\left(\sin\left(\frac{\pi}{2}-y\right)\right)^{\tan\left(\frac{\pi}{2}-y\right)}$$

$$=\lim_{y\to 0}(\cos y)^{\cot y}$$

$$=\lim_{y\to 0}(1+(\cos y-1))^{\cot y}$$

$$=\lim_{y\to 0}\left(1-2\sin^2\left(\frac{y}{2}\right)\right)^{\cot y}$$

$$=\lim_{y\to 0}\left(1-2\sin^2\left(\frac{y}{2}\right)\right)^{\frac{\cos y}{\sin y}}$$

$$=e^{\lim_{y\to 0}\ln\left(1-2\sin^2\left(\frac{y}{2}\right)\right)^{\frac{\cos y}{\sin y}}}$$

$$=e^{\lim_{y\to 0}\frac{\cos y}{\sin y}\times\ln\left(1-2\sin^2\left(\frac{y}{2}\right)\right)}$$

$$=e^0=1$$

$$\lim_{x\to\frac{\pi}{2}}(\sin x)^{\tan x}=1$$

نهاية 32: $\lim_{x\to 0}\frac{2^{\sec x}-2^{\cos x}}{x^2}$.

نعلم أن: $\lim_{x\to 0}\frac{e^x-1}{x}=1$.

$$\lim_{x\to 0}\frac{2^{\sec x}-2^{\cos x}}{x^2}=\lim_{x\to 0}\frac{e^{\ln 2.\sec x}-e^{\ln 2.\cos x}}{x^2}$$

$$=\lim_{x\to 0}\frac{e^{\ln 2.\cos x}\left(e^{\ln 2\frac{1}{\cos x}.\ln 2.\cos x}-1\right)}{x^2}$$

$$=\lim_{x\to 0}\frac{e^{\ln 2.\cos x}\left(e^{\ln 2\left(\frac{1-\cos^2 x}{\cos x}\right)}-1\right)}{\ln 2\left(\frac{\sin^2 x}{\cos x}\right)}\times\frac{\ln 2\left(\frac{\sin^2 x}{x^2}\right)}{\cos x}$$

$$=2\ln 2$$

إذن: $\lim_{x\to 0}\frac{2^{\sec x}-2^{\cos x}}{x^2}=2\ln 2$.

نهاية 33: $\lim_{x\to 0}\frac{1}{x\sin x}-\frac{1}{x^2}$.

نعلم أن: $\lim_{x\to 0}\frac{x-\sin x}{x^3}=\frac{1}{6}$.

$$\lim_{x\to 0}\frac{1}{x\sin x}-\frac{1}{x^2}=\lim_{x\to 0}\frac{x-\sin x}{x^2\sin x}$$

$$=\lim_{x\to 0}\frac{x-\sin x}{x^3}\times\frac{x^3}{x^2\sin x}$$

$$=\lim_{x\to 0}\frac{x-\sin x}{x^3}\times\frac{x}{\sin x}$$

$$=\frac{1}{6}.1=\frac{1}{6}$$

و بالتالي: $\lim_{x\to 0}\frac{1}{x\sin x}-\frac{1}{x^2}=\frac{1}{6}$.

نهاية 34: $\lim_{x\to e}\frac{x-e\ln x}{(x-e)^2}$

ليكن: $\lim_{x\to e}\frac{x-\ln x-1}{(x-1)^2}=\frac{1}{2}$.

بوضع: $x=ey$
$x\to e, y\to 1$

$$\lim_{x\to e}\frac{x-e\ln x}{(x-e)^2}=\lim_{y\to 1}\frac{ey-e\ln ey}{(ey-e)^2}$$

$$=\lim_{y\to 1}\frac{e}{e^2}\times\frac{(y-\ln e-\ln y)}{(y-1)^2}$$

$$= \frac{1}{e} \times \lim_{y \mapsto 1} \frac{(y - 1 - \ln y)}{(y-1)^2}$$

$$= \frac{1}{e} \cdot \frac{1}{2}$$

$$= \frac{1}{2e}$$

ومنه: $\lim_{x \mapsto e} \frac{x - e\ln x}{(x-e)^2} = \frac{1}{2e}$.

نهاية 35 : $\lim_{x \mapsto 0} \frac{x - \tan^{-1} x}{x^3}$

نعلم أن: $\lim_{x \mapsto 0} \frac{x - \sin x}{x^3} = \frac{1}{6}$.

نضع: $y = \tan^{-1} x$.

$$\lim_{x \mapsto 0} \frac{x - \tan^{-1} x}{x^3} = \lim_{y \mapsto 0} \frac{\tan y - y}{\tan^3 y}$$

$$= \lim_{y \mapsto 0} \frac{\sin y - y.\cos y}{y^3 \cos y}$$

$$= \lim_{y \mapsto 0} \frac{(\sin y - y) + (y - y.\cos y)}{y^3} \times \frac{1}{\cos y}$$

$$= \lim_{y \mapsto 0} \frac{(\sin y - y)}{y^3} \times \frac{1}{\cos y} + \lim_{y \mapsto 0} \frac{\left(y.2\sin^2\left(\frac{y}{2}\right)\right)}{y^3} \times \frac{1}{\cos y}$$

$$= \frac{-1}{6}.1 + 2.\frac{1}{4} = \frac{1}{3}$$

و منه $\lim_{x \mapsto 0} \frac{x - \tan^{-1} x}{x^3} = \frac{1}{3}$.

نهاية 36: $\lim_{x \mapsto 0} \frac{1 - \cos(3x)}{1 - \cos(4x)}$.

$$\lim_{x \mapsto 0} \frac{1 - \cos(3x)}{1 - \cos(4x)} = \lim_{x \mapsto 0} \frac{2\sin^2\left(\frac{3x}{2}\right)}{2\sin^2\left(\frac{4x}{2}\right)}$$

$$= \lim_{x \mapsto 0} \frac{\sin^2\left(\frac{3x}{2}\right)}{\sin^2(2x)} = \frac{9}{16}$$

ومنه: $\lim_{x \mapsto 0} \frac{1 - \cos(3x)}{1 - \cos(4x)} = \frac{9}{16}$.

نهاية 37: $\lim_{x \mapsto 1} \frac{x\sqrt{x+3} - 2}{x^2 + x - 2}$.

$$\lim_{x \mapsto 1} \frac{x\sqrt{x+3} - 2}{x^2 + x - 2}$$

$$= \lim_{x \mapsto 1} \frac{x\sqrt{x+3} - \sqrt{x+3} + \sqrt{x+3} - 2}{(x-2)(x-1)}$$

$$= \lim_{x \mapsto 1} \frac{(x-1)\sqrt{x+3}}{(x-2)(x-1)} + \lim_{x \mapsto 1} \frac{(x+3)^{\frac{1}{2}} - 4^{\frac{1}{2}}}{(x+3-4)} \cdot \frac{1}{x+2}$$

$$= \frac{2}{3} + \frac{1}{12} = \frac{3}{4}$$

اذن: $\lim_{x \mapsto 1} \frac{x\sqrt{x+3} - 2}{x^2 + x - 2} = \frac{3}{4}$.

نهاية 38: $\lim_{x \mapsto \frac{\pi}{4}} \frac{\tan x - 1}{\left(x - \frac{\pi}{4}\right)}$.

نعلم أن: $\tan a - \tan b = \frac{\sin(a-b)}{\cos a.\cos b}$.

$$\lim_{x \mapsto \frac{\pi}{4}} \frac{\tan x - 1}{\left(x - \frac{\pi}{4}\right)} = \lim_{x \mapsto \frac{\pi}{4}} \frac{\tan x - \tan\left(\frac{\pi}{4}\right)}{\left(x - \frac{\pi}{4}\right)}$$

$$\lim_{x \mapsto \frac{\pi}{4}} \frac{\sin\left(x - \frac{\pi}{4}\right)}{\left(x - \frac{\pi}{4}\right)\cos x.\cos\left(\frac{\pi}{4}\right)}$$

$$\lim_{x\to\frac{\pi}{4}} \frac{\sin\left(x-\frac{\pi}{4}\right)}{\left(x-\frac{\pi}{4}\right)} \times \frac{1}{\cos x.\cos\left(\frac{\pi}{4}\right)}$$

$$=1.\frac{1}{\frac{\sqrt{2}}{2}}.\frac{1}{\frac{\sqrt{2}}{2}}=2$$

و بالتالي: $\lim_{x\to\frac{\pi}{4}} \frac{\tan x-1}{\left(x-\frac{\pi}{4}\right)}=2$.

نهاية 39: $\lim_{x\to0} \frac{\ln\left(2x^2+\cos(2x)\right)}{x^4}$.

$$\lim_{x\to0} \frac{\ln\left(2x^2+\cos(2x)\right)}{x^4}$$

$$=\lim_{x\to0} \ln\left(2x^2+\cos(2x)\right)^{\frac{1}{x^4}}$$

$$=\lim_{x\to0} \ln\left(2+2x^2+\cos(2x)-2\right)^{\frac{1}{x^4}}$$

$$=\ln e^{\lim_{x\to0}\frac{\left(2x^2-2\sin^2 x\right)}{x^4}}$$

$$=\ln e^{\lim_{x\to0} 2\frac{(x-\sin x)}{x^4}.\frac{(x+\sin x)}{x}}$$

$$=\frac{2}{3}$$

و منه: $\lim_{x\to0} \frac{\ln\left(2x^2+\cos(2x)\right)}{x^4}=\frac{2}{3}$.

نهاية 40: $\lim_{x\to\frac{\pi}{6}} \frac{\sin(x)-\frac{1}{2}}{6x-\pi}$.

$$\lim_{x\to\frac{\pi}{6}} \frac{\sin(x)-\frac{1}{2}}{6x-\pi}=\lim_{x\to\frac{\pi}{6}} \frac{\sin(x)-\sin\left(\frac{\pi}{6}\right)}{6\left(x-\frac{\pi}{6}\right)}$$

$$=\lim_{x\to\frac{\pi}{6}} \frac{2\sin\left(\frac{\left(x-\frac{\pi}{6}\right)}{2}\right).\cos\left(\frac{\left(x+\frac{\pi}{6}\right)}{2}\right)}{6\left(x-\frac{\pi}{6}\right)}$$

$$=\lim_{x\to\frac{\pi}{6}} \frac{2\sin\left(\frac{\left(x-\frac{\pi}{6}\right)}{2}\right).}{\left(x-\frac{\pi}{6}\right)} \times \lim_{x\to\frac{\pi}{6}} \frac{\cos\left(\frac{\left(x+\frac{\pi}{6}\right)}{2}\right)}{6}$$

$$=2.\frac{1}{2}.\frac{\frac{\sqrt{3}}{2}}{6}=\frac{\sqrt{3}}{12}$$

و بالتالي: $\lim_{x\to\frac{\pi}{6}} \frac{\sin(x)-\frac{1}{2}}{6x-\pi}=\frac{\sqrt{3}}{12}$.

نهاية 41: $\lim_{x\to0} \frac{\cos x-\sqrt[3]{\cos x}}{\sin^2 x}$.

$$\lim_{x\to0} \frac{\cos x-\sqrt[3]{\cos x}}{\sin^2 x}=\lim_{x\to0} \frac{\left(\cos(x)-1\right)-\left(\sqrt[3]{\cos x}-1\right)}{\sin^2 x}$$

$$=-1\times\lim_{x\to0} \frac{\left(\cos(x)-1\right)-\left(\sqrt[3]{\cos x}-1\right)}{\cos^2(x)-1}$$

$$=-1\times\lim_{x\to0} \frac{\left(\cos(x)-1\right)}{\cos^2(x)-1}+\lim_{x\to0} \frac{\left(\sqrt[3]{\cos x}-1\right)}{\cos^2(x)-1}$$

$$=\frac{-1}{3}$$

و بالتالي: $\lim_{x\to0} \frac{\cos x-\sqrt[3]{\cos x}}{\sin^2 x}=\frac{-1}{3}$.

نهاية 42: $\lim\limits_{x\mapsto 0}\dfrac{\ln\left(\dfrac{3-\cos 2x-2x^2}{2}\right)}{x^4}$

$$\lim_{x\mapsto 0}\frac{\ln\left(\dfrac{3-\cos 2x-2x^2}{2}\right)}{x^4}$$

$$=\lim_{x\mapsto 0}\ln\left(\frac{3-\cos 2x-2x^2}{2}\right)^{\frac{1}{x^4}}$$

$$=\lim_{x\mapsto 0}\ln\left(\frac{2}{2}+\frac{1-\cos 2x-2x^2}{2}\right)^{\frac{1}{x^4}}$$

$$=\lim_{x\mapsto 0}\ln\left(1+\sin^2 x-x^2\right)^{\frac{1}{x^4}}$$

$$=\ln\lim_{x\mapsto 0}e^{\left(\frac{\sin x-x}{x^3}\cdot\frac{\sin x+x}{x}\right)}$$

$$=\ln e^{\frac{-2}{6}}=\frac{-1}{3}$$

إذن: $\lim\limits_{x\mapsto 0}\dfrac{\ln\left(\dfrac{3-\cos 2x-2x^2}{2}\right)}{x^4}=\dfrac{-1}{3}$.

نهاية 43: $\lim\limits_{x\mapsto\sqrt{\pi}}\dfrac{1+\cos\sqrt{\pi}x}{\pi-x^2}$.

$$\lim_{x\mapsto\sqrt{\pi}}\frac{1+\cos\sqrt{\pi}x}{\pi-x^2}=\lim_{x\mapsto\sqrt{\pi}}\frac{2\cos^2\left(\dfrac{\sqrt{\pi}x}{2}\right)}{\pi-x^2}$$

$$=\lim_{x\mapsto\sqrt{\pi}}\frac{2\sin^2\left(\dfrac{\pi}{2}-\dfrac{\sqrt{\pi}x}{2}\right)}{\pi-x^2}$$

$$=\lim_{x\mapsto\sqrt{\pi}}2\left(\frac{\sin\left(\dfrac{\sqrt{\pi}(\sqrt{\pi}-x)}{2}\right)}{\sqrt{\pi}-x}\right)\times\left(\frac{\sin\left(\dfrac{\sqrt{\pi}(\sqrt{\pi}-x)}{2}\right)}{\sqrt{\pi}+x}\right)$$

$$=2\times\frac{\sqrt{\pi}}{2}\times 0=0$$

و منه: $\lim\limits_{x\mapsto\sqrt{\pi}}\dfrac{1+\cos\sqrt{\pi}x}{\pi-x^2}=0$.

نهاية 44: $\lim\limits_{x\mapsto\frac{\pi}{6}}\dfrac{1-\sqrt{3}\tan x}{(\pi-6x)}$

نعلم أن: $\tan a-\tan b=\dfrac{\sin(a-b)}{\cos a.\cos b}$.

$$\lim_{x\mapsto\frac{\pi}{6}}\frac{1-\sqrt{3}\tan x}{(\pi-6x)}=\lim_{x\mapsto\frac{\pi}{6}}\frac{\sqrt{3}}{6}\left(\frac{\dfrac{1}{\sqrt{3}}-\tan x}{\left(\dfrac{\pi}{6}-x\right)}\right)$$

$$=\frac{\sqrt{3}}{6}\lim_{x\mapsto\frac{\pi}{6}}\left(\frac{\dfrac{1}{\sqrt{3}}-\tan x}{\left(\dfrac{\pi}{6}-x\right)}\right)$$

$$=\frac{\sqrt{3}}{6}\lim_{x\mapsto\frac{\pi}{6}}\left(\frac{\tan\left(\dfrac{\pi}{6}\right)-\tan x}{\left(\dfrac{\pi}{6}-x\right)}\right)$$

$$=\frac{\sqrt{3}}{6}\lim_{x\mapsto\frac{\pi}{6}}\left(\frac{\dfrac{\sin\left(\dfrac{\pi}{6}-x\right)}{\cos\dfrac{\pi}{6}.\cos x}}{\left(\dfrac{\pi}{6}-x\right)}\right)$$

$$=\frac{2\sqrt{3}}{9}$$

إذن: $\lim\limits_{x\mapsto\frac{\pi}{6}}\dfrac{1-\sqrt{3}\tan x}{(\pi-6x)}=\dfrac{2\sqrt{3}}{9}$.

نهاية 45: $\lim\limits_{x\mapsto\infty}\left(\dfrac{x^2+5x+4}{x^2-3x+7}\right)^x$

$$\lim_{x\mapsto\infty}\left(\frac{x^2+5x+4}{x^2-3x+7}\right)^x=\lim_{x\mapsto\infty}\left(1+\frac{8x-3}{x^2-3x+7}\right)^x$$

$$=e^{\lim_{x\mapsto\infty}\left(\frac{8x-3}{x^2-3x+7}\right).x}=e^8$$

اذن: $\lim_{x\mapsto\infty}\left(\frac{x^2+5x+4}{x^2-3x+7}\right)^x=e^8$.

نهاية 46: $\lim_{x\mapsto\infty}\left(\cos\left(\frac{x}{m}\right)\right)^m$.

نعلم أن: $\lim_{x\mapsto1}\left(\frac{\ln x}{x-1}\right)=1$

بوضع : $m=\frac{1}{n}$ نجد:

$$\lim_{m\mapsto\infty}\left(\cos\left(\frac{x}{m}\right)\right)^m=\lim_{n\to0}\left(\cos(nx)\right)^{\frac{1}{n}}$$

$$=e^{\lim_{n\to0}\frac{l\,n(\cos(nx))}{n}}$$

$$=e^{\lim_{n\to0}\frac{l\,n(\cos(nx))}{\cos(nx)-1}\times\frac{\cos(nx)-1}{n}}$$

$$=e^{\lim_{n\to0}\frac{-2\sin^2(nx)}{n}}$$

$$=e^{\lim_{n\to0}\frac{\sin(nx)}{n}\times(-2\sin(nx))}$$

$$=e^0=1$$

اذن: $\lim_{x\mapsto\infty}\left(\cos\left(\frac{x}{m}\right)\right)^m=1$.

نهاية 47: $\lim_{x\mapsto e}\frac{\ln x-1}{(x-e)}$.

نعلم أن: $\lim_{x\mapsto1}\left(\frac{\ln x}{x-1}\right)=1$

$\lim_{x\mapsto0}\left(\frac{\ln(x+1)}{x}\right)=1$

و $\lim_{x\mapsto0}\left(\frac{e^x-1}{x}\right)=1$.

$$\lim_{x\mapsto e}\frac{\ln x-1}{(x-e)}=\lim_{x\mapsto e}\frac{\ln x-1}{e\left(\frac{x}{e}-1\right)}$$

$$\lim_{x\mapsto e}\frac{\ln\left(\frac{x}{e}\right)}{e\left(\frac{x}{e}-1\right)}=\frac{1}{e}.1=\frac{1}{e}$$

اذن: $\lim_{x\mapsto e}\frac{\ln x-1}{(x-e)}=\frac{1}{e}$.

نهاية 48: $\lim_{x\mapsto\frac{\pi}{3}}\frac{\sin x-\sqrt{3}\cos x}{4x-\frac{x\pi}{3}}$.

نعلم أن: $\tan x-\tan y=\frac{\sin(x-y)}{\cos x.\cos y}$.

$$\lim_{x\mapsto\frac{\pi}{3}}\frac{\sin x-\sqrt{3}\cos x}{4x-\frac{4\pi}{3}}$$

$$=\lim_{x\mapsto\frac{\pi}{3}}\cos(x)\times\lim_{x\mapsto\frac{\pi}{3}}\frac{\tan x-\sqrt{3}}{4x-\frac{4\pi}{3}}$$

$$=\frac{1}{2}\times\lim_{x\mapsto\frac{\pi}{3}}\frac{\tan x-\tan\frac{\pi}{3}}{4\left(x-\frac{\pi}{3}\right)}$$

$$=\frac{1}{2}\times\lim_{x\mapsto\frac{\pi}{3}}\frac{\frac{\sin\left(x-\frac{\pi}{3}\right)}{\cos x.\cos\frac{\pi}{3}}}{4\left(x-\frac{\pi}{3}\right)}$$

$$= \frac{1}{2} \times \lim_{x \mapsto \frac{\pi}{3}} \frac{\dfrac{\sin\left(x-\frac{\pi}{3}\right)}{\cos x . \cos\frac{\pi}{3}}}{4\left(x-\frac{\pi}{3}\right)}$$

$$= \frac{1}{2} \times \lim_{x \mapsto \frac{\pi}{3}} \frac{s\text{in}\left(x-\frac{\pi}{3}\right)}{4\left(x-\frac{\pi}{3}\right)} \times \frac{1}{\cos x . \cos\frac{\pi}{3}}$$

$$= \frac{1}{2}$$

اذن: $\lim_{x \mapsto \frac{\pi}{3}} \dfrac{\sin x - \sqrt{3}\cos x}{4x - \frac{x\pi}{3}} = \dfrac{1}{2}$.

نهاية 49: $\lim_{x \mapsto a} \dfrac{x^m - a^m}{x^n - a^n}$

$$\lim_{x \mapsto a} \frac{x^m - a^m}{x^n - a^n} = \frac{m}{n}(a)^{m-n}$$

نهاية 50: $\lim_{x \mapsto 0} \dfrac{1-\sqrt[6]{\cos 2x}}{x^2}$.

نعلم أن : $\lim_{x \mapsto a} \dfrac{x^m - a^m}{x^n - a^n} = \dfrac{m}{n}(a)^{m-n}$

$$\lim_{x \mapsto 0} \frac{1-\sqrt[6]{\cos 2x}}{x^2}$$

$$= \lim_{x \mapsto 0} \frac{1-\sqrt[6]{\cos 2x}}{1-\cos 2x} \times \frac{1-\cos 2x}{x^2}$$

$$= \lim_{x \mapsto 0} \frac{1-(\cos 2x)^{\frac{1}{n}}}{1-\cos 2x} \times \frac{1-\cos 2x}{x^2}$$

$$= \frac{\frac{1}{n}}{n} .(1)^{\frac{1}{n}-1} . \lim_{x \mapsto 0} \frac{\sin^2 x}{x^2} = \frac{1}{3}$$

اذن: $\lim_{x \mapsto 0} \dfrac{1-\sqrt[6]{\cos 2x}}{x^2} = \dfrac{1}{3}$

ملاحظة

طريقة حل النهايات ليست وحيدة

تجدون هذا الملف

5min Maths